# Controlled 3D Biped Stepping Animations using the Inverted Pendulum and Impulse Constraints

Ben Kenwright
*Newcastle University, United Kingdom*
*b.kenwright@ncl.ac.uk*

***Abstract*—This paper presents a method for generating intelligent upright biped stepping motions for real-time dynamic environments. Our approach extends the inverted pendulum (IP) model by means of an impulse-based technique to achieve rigid-leg constraints during foot support transitions. The impulse-based method in cooperation with the IP method provides a computationally fast, straightforward, and robust solution for achieving stiff-knee joints that are desired during casual stepping motions, such as standing and walking. Furthermore, we demonstrate how the impulse-based inverted pendulum (IIP) model can be extended to embody rotational information to synthesize more dynamic actions, such as when the feet leave the ground or when slipping (i.e., foot friction).**

***Keywords*-Character Animation; Balancing; Real-Time; Responsive; Inverted Pendulum; Biped; Reactive; Avatar; Impulse; Stepping**

## I. Introduction

Generating physically accurate responsive character motions in real-time for dynamic environments is challenging and interesting [1]–[5]. Employing physics-based methods with intelligent controllers enables us to produce characters that go beyond static ragdoll-like puppets with hard-coded data-driven key-framed libraries towards more life-like and realistic solutions. However, constructing a reliable, real-time, flexible controller that generates controllable interactive character motions can be complex and difficult. In this paper, we demonstrate how we can use an impulse-based method in combination with the Inverted Pendulum (IP) model [6] to produce intelligent upright stepping information to create controlled, responsive, biped animations.

The process of placing our feet intelligently at crucial positions to achieve a desired outcome (e.g., turning on the spot or walking up stairs) is fundamental for any virtual biped character. We propose employing a low-dimensional controller for the creation of crucial stepping and body information to produce upright human movements.

The IP controller is a physics-based model with intelligent foot placement logic. Furthermore, since it is a physics-based model, it incorporates real-world properties, such as mass and velocity, to control the character's responsive stepping. We use the IP model as a fundamental building block for generating realistic, responsive, and dynamic biped animations.

Our method focuses on upright locomotion, balancing, and responsive stepping through the exploitation of the IP controller's continuous feedback mechanism for the self-correction of balancing errors. For example, if we apply a push force disturbance to the character, then the IP will automatically calculate a corrective step to remedy the problem and recover from the disturbance. Our system generates the biped stepping animation from the IP model by extracting key information (i.e., foot placement and postural location) and combines them with an uncomplicated inverse kinematic (IK) solver to generate full-body skeletal poses.

The Impulse-based Inverted Pendulum (IIP) technique is the root of our system for generating motions for a full-body biped character by continuously analyzing and extrapolating trajectory information for end-effectors (e.g., feet and pelvis) to create the character's animation. The IIP model is an uncomplicated, computationally efficient, and logical approach for correcting and emulating a virtual human's balancing and walking movements.

**Motivation:** The motivation behind this work focuses on moving towards more intelligent procedural physics-based techniques for generating dynamic natural responsive character animations and away from traditional key-framed data-driven methods. For example, exploiting procedural controller mechanisms that use intelligent motion driven logic algorithms to create characters that place their feet based on comfort and balancing reasoning.

**Contribution:** The contribution of this paper is the demonstration and evaluation of the Impulse-based Inverted Pendulum (IIP) technique as a method for generating balanced upright motions for biped characters. Furthermore, we show how impulses can incorporate friction when added to the basic IP model to represent foot slipping. We also extend the point mass body to include orientation (i.e., an elongated rigid body) to obtain additional features (e.g., when the feet leave the floor, the feet will follow the angular trajectory of the body). In summary, the important points that we address in this paper that are novel and interesting are:

- Impulses in conjunction with the IP model
- Controlled balanced stepping (e.g., the ability to maintain a controlled steered velocity during disturbances and/or while pushing/pulling an object)
- Real-time stepping on unstable movable terrain (e.g., a bridge)
- Impulse IP with orientated body (see Figure 2)
- Friction and slipping (using an orientated rigid body instead of a point mass)

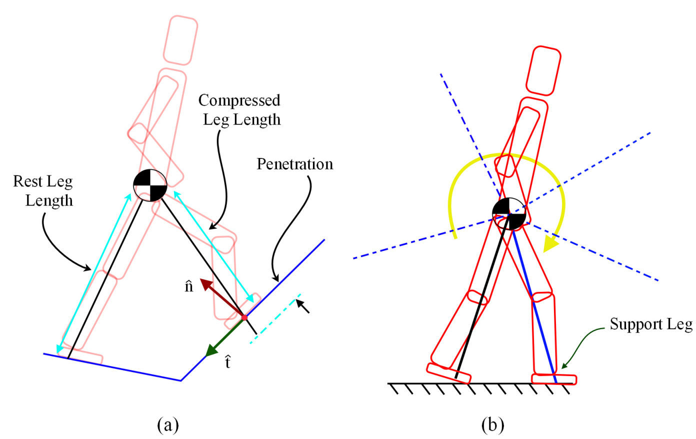



Figure 1. **Rigid Leg Penetration Solution** - Adding a corrective bias to the penetration depth to simulate a bent knee gradually being straightened.

## II. Related Work

There has been a tremendous amount of research into generating and modifying character-based animation methods to produce more realistic, extendable, and interactive characters.

In particular, since classical mechanics is the study of the motion of bodies under forces through physics-based laws. These physics-based laws can emulate the real-world properties of a character (i.e., mass, muscle strength, frictional forces) and hence, provide a method of synthesizing real-world movement. Consequently, it should come as no shock that numerous researchers have investigated and explored physics-based techniques in conjunction with controller-based approaches to produce more dynamic and interactive characters [3], [7]–[11].

Additionally, hybrid physics-based methods have exploited local optimization techniques to generate new animations, which could handle unforeseen situations as shown by Stewart and Cremer [12], [13]. Then again, while physics-based methods can generate dynamic motions, the modification of existing hard-coded animation libraries through data-driven methods shows promise in some cases where artistic control is required. Hence, a particular area of research has been into developing data-driven techniques that modify large

Journal: Cyberworlds International Conference 2013, 21-23 October, Yokohama
Author: Ben Kenwright
Title: Controlled 3D Biped Stepping Animations using the Inverted Pendulum and Impulse Constraints

collections of animation data so the final character motions are more dynamic and responsive (i.e., corrective stepping motions) [14], [15].

However, this paper and the motivation behind this work is aimed at creating life-like responsive motions using a simplified approximation model without data (i.e., pre-canned animations). For example, the work by Panne [16] demonstrated how character motions could be generated using only foot placement information. While, Jain et al. [17], used a similar method with a support polygon to position and control the center-of-mass so it remained balanced during stepping. Focusing purely on stepping, there have also been other physics-based methods that have used stepping methods as a primary system for achieving upright character motions [1], [18].

Nevertheless, we use the inspiring work by Kajita et al. [19], and Sugihara [20], as the driving direction for this paper's work, since it has been proven from an early stage that the IP model possesses the potential for mimicking upright human motions. The IP is a biomechanically inspired approach that synthesizes how we, as humans, perform balancing and upright locomotion, since the concept of intelligent foot placement is crucial for balancing during stepping and walking and has long been explored in biomechanics and robotics [21]–[23].

Furthermore, Pratt [24], [25] confirmed how the IP could be used as a viable method for generating stable responsive biped humanoids. Moreover, Kenwright [26] recently showed an extension to the IP model for generating real-time responsive stepping and upper-body postural information for characters by using an elongated body instead of a single-point mass. Later, Kenwright [5] also demonstrated a stable and controllable solution by adding an ankle feedback approximation to overcome some of the oversimplifications of the IP model (i.e., constantly stepping, better steering control).

This paper incorporates foot slipping from friction by using the approach put forward by Pratt et al. [25] who extended the IP model with a flywheel and the angular momentum model by Komura et al. [2]; who both extended the point-mass IP model to include angular considerations. Similarly, Wu and Zordan [27], performed controlled stepping by incorporating momentum.

In summary, our approach builds upon the popular IP model, which provides a simplified dynamic model for generating crucial balancing information. For example, the IP model has been exploited time-and-time again due to its computationally efficiency and simplicity [4], [10], [28], [29]. However, we combine the IP model with an impulse-based method, since it enables us to enforce constraints efficiently without hindering the IP model's speed or robustness for generating a flexible controllable stepping solution.

## III. The Inverted Pendulum (IP)

In this section, we describe the underlying principles of the inverted pendulum (IP) that we use to generate responsive dynamic character stepping motions.

The IP is an intuitive, abstract, and flexible approach for identifying and generating balancing information. Furthermore, it can be used for different motion styles and simplifies the multi-segment skeleton down so that we do not need to perform complex kinematic and dynamic calculations. The IP works just as well in 2D as 3D and is efficient enough to run at real-time frame rates. The simplest form of the IP is a single-point mass suspended by a massless rod (i.e., a leg). The massless leg has a pin-point contact with the terrain (e.g., the foot support region is a single-point when one foot is in contact with the ground or a line when two feet are in contact).

In essence, the IP model provides a low-dimensional description of the fundamental characteristics of a character's movement (e.g., center-of-mass trajectory) during upright stepping. Furthermore, *we can alter certain parameters, for example, the angle of the step, the duration, or location to produce a wide range of stepping actions for different upright motions.*

### A. Spring Loaded Inverted Pendulum (SLIP)

The SLIP model has been used to generate reliable stance information for numerous low-dimensional models [4], [26]. The spring-damper system emulates how the leg flexes and bends. For example, Equation 1 shows the basic equation to generate the responsive force between the foot and the ground.

$$F = (x - l)k_s - (v \cdot \hat{n})k_d \quad (1)$$

where $F$ is the force applied to the body from the foot, $x$ is the current length from the center-of-mass (CoM) to the foot contact, $l$ is the leg rest length; $v$ is the relative velocity between the body CoM and the ground; $\hat{n}$ is the normal direction from the foot contact to the CoM, and $k_s$, $k_d$ are the spring and damper coefficients. The dot product of the normal and velocity ensure the axis alignment of the feedback is only along the leg and does not affect the tangential movement. If the ground is fixed and immovable, the relative velocity $v$ is the velocity of the CoM.

### B. Impulse Inverted Pendulum (IIP)

On occasions, we want a character's legs to be incompressible (i.e., stiff knee joints). Impulse forces allow us to accomplish this using a computationally fast and algorithmically straightforward way. We discuss and present the impulse equations in Section III-E.

### C. Inverted Pendulum Wheel

A novel, simple, and easy method of visualizing and understanding how the inverted pendulum works in its fundamental state is to look at the feet and body as a wheel with spokes. This enables the reader to see clearly how the inverted pendulum functions to generate the foot placement information, as shown in Figure 1(b).

The wheel model is just to visualize the fundamental principle. The angle between each foot placement is dynamically changed depending upon the desired outcome. For example, if we want the character to speed-up, slow-down or come to a stop.

### D. Body Orientation

The basic IP model represents the character's body as a single-point mass with no rotational information. This absence of rotational information can hide crucial dynamic motion characteristics (e.g., the angular momentum when jumping and spinning).

Following the novel flywheel method by Pratt et al. [24], we extend the point-mass body to include rotational information so that we can represent upper-body dynamics from the arms, torso, and feet slipping due to friction. Furthermore, by adding orientation to the body, we are able to simulate dynamic flight behaviors. For example, when the feet leave the ground during leaps and rolls, the body rotational information would present us with angular dynamics (e.g., angular momentum).

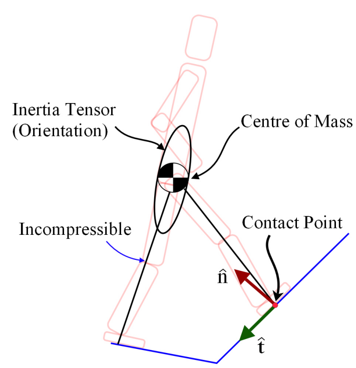



Figure 2. **Orientation & Angular Velocity** - We can extend the basic IP point-mass model to use an elongated rigid body to provide additional information about the angular velocity and orientation change. For example, the figure shows the contact point of the foot on uneven terrain.

### E. Impulse Equations

Rigid body dynamics have employed impulse-based methods in the past due to their simplicity, computational efficiency, and robustness [30] to respond to collisions and achieve constraints [31]. Hence, we take advantage of the impulse-based method and exploit it in the IP based technique to acquire the same advantages and produce a stable and efficient method for producing stepping motions with rigid incompressible leg movements (i.e., stiff knees).

The impulse response for a pair of bodies in contact is given in Equations 2 to Equation 4. The equations are for six-degrees of freedom (i.e., linear and angular components). Then again, we can simplify the equations to remove the angular component for less complicated simulations when we are only concerned with translational movement (e.g., point-mass IP).

$$j = \frac{-v_r(1+e)}{\frac{1}{m_1} + \frac{1}{m_2} + \hat{n} \cdot \left(\frac{r_1 \times \hat{n}}{I_1} \times r_1\right) + \hat{n} \cdot \left(\frac{r_2 \times \hat{n}}{I_2} \times r_2\right)} \quad (2)$$

where $j$ scalar impulse, $v_r$ relative velocity, $e$ coefficient of restitution, $n$ is the unit length contact normal, $m_1$, $m_2$ mass of the rigid bodies, $I_1$, $I_2$ inertia tensors for the rigid bodies, $r_1$, $r_2$ vectors from the center-of-mass to the point of contact for the rigid bodies.

The rigid bodies' angular and linear velocities are modified using Equation 3.

$$\begin{aligned} v_{1,2} &= v_{1,2} +/- \frac{[j\hat{n} + \mu j \hat{t}]}{I_{1,2}} \\ \omega_{1,2} &= \omega_{1,2} +/- \frac{r_{1,2} \times [j\hat{n} + \mu j \hat{t}]}{I_{1,2}} \end{aligned} \quad (3)$$

where $v_{1,2}$ is the linear velocity of the rigid body, $\omega_{1,2}$ is the angular velocity of the rigid body, $\mu$ is the coefficient of friction, $\hat{t}$ is the unit length tangential normal of the contact point and the $+/-$ represents the sign for body 1 or body 2. We calculate the tangential component using Equation 4.

$$\hat{t} = \frac{(\hat{n} \times v_r) \times \hat{n}}{||(\hat{n} \times v_r) \times \hat{n}||} \quad (4)$$

We have implemented the impulse equation between two movable bodies, since a character can be walking on a surface that is movable (for example, as shown in Figure 4(a) the model walking across a bridge). For an in depth detailed explanation of impulse-based methods, see the excellent work from Baraff [32], or Mirtich [30].

### F. *Penetration Depth (Knee Bent)*

For occasions when the foot placement results in the leg-length being less than the rest length, then we resolve this by adding a bias to the impulse magnitude, as shown in Equation 5.

$$j = j + (l_{rest} - l_{actual})k_p \quad (5)$$

where $l_{rest}$, $l_{actual}$ are the leg-lengths at rest and currently, and $k_p$ is the bias factor for how fast we want our leg to be straightened (typically 0.1 to 5).

### G. *Inverted Pendulum Step-Distance*

The fundamental IP model works by pole vaulting the point-mass over the support-leg during walking while transitioning instantly between the left and right leg between stepping. Hence, energy is constantly converted between kinetic and potential energy to generate constant perpetual stepping motions through the conservational law of mechanical energy (i.e., as shown by the fundamental Equation 6, whereby the sum of the potential and kinetic energy should be conserved before and after the stepping phase). This principle can be used to estimate the character's foot placement position (e.g., step length) to control the speed (e.g., walk faster or come to a stop) and direction while maintaining balance, see Coros et al. [28] and Tsai et al. [29].

$$0.5mgh + 0.5mv^2 = 0.5mgh' + 0.5mv'^2 \quad (6)$$

where $m$ is the mass, $g$ is the gravitational constant (i.e., $9.8m/s^2$), $h$ and $h'$ are the heights before and after the stepping phase, $v$ and $v'$ are the velocities before and after the stepping phase.

From which we develop the Equation 7 with reference to Figure 3(b).

$$\begin{aligned} v' &= 0 \\ h' &= L = \sqrt{((h-s)^2 + d^2)} \text{ (Sub in Eq. 6)} \\ \frac{1}{2}mv^2 + mg(h-s) &= mg(\sqrt{((h-s)^2 + d^2)}) \text{ (Solve for d)} \\ d &= \frac{v}{2g}\sqrt{(v^2 + 4g(h-s))} \end{aligned} \quad (7)$$

We define the conditions for the IP model as either halting or moving in a specific direction (i.e., $(\hat{u} \cdot \hat{a})||F_g|| \geq (\hat{b} \cdot F_p)$, as shown in Figure 3(a)). We integrate in an approximate bias factor $\beta$, which we add to the stepping distance, as shown in Equation 8 to include a pull (or push) force in the stepping distance to maintain a controllable upright velocity in the desired direction. For example, if a constant

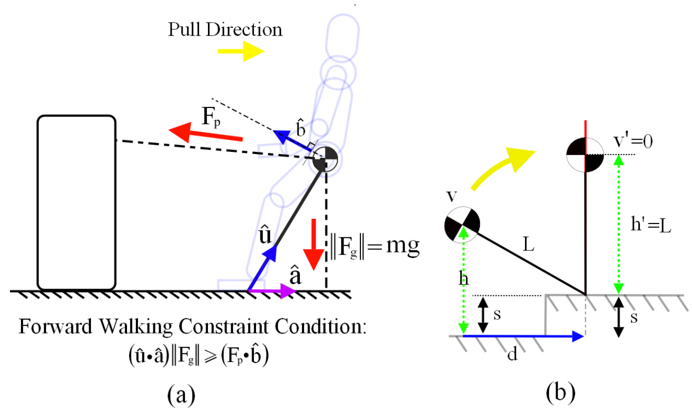



Figure 3. **Controlled Stepping Distance Formulation** - Extending the basic model to include deviation for changes in terrain height (e.g., uneven terrain, such as stairs or sloping ground).

pull force is applied to the character, it will be able to maintain a constant stepping speed in the desired direction by means of small constant steps. Without this additional approximation, it will speed up and slow down and take forward and backward balancing steps.

$$\begin{aligned} \beta &\geq \frac{(\hat{b} \cdot F_p)}{(\hat{u} \cdot \hat{a})||F_g||} \\ d' &= d + \beta k_p \end{aligned} \quad (8)$$

where $k_p$ is a scaling factor (e.g., 0.4), $d$ is the stepping distance calculated using Equation 7, $F_p$ is the external pull or push force exerted on the body, and $\hat{u}, \hat{a}, \hat{b}$ are unit vectors as shown in Figure 3(a). Hence, we modify the stepping distance Equation 7 so that it can continue to remain balanced while walking under the influence of push or pull forces (e.g., while dragging a box) as shown above in Equation 8. Whereby, the pull (or push) force reduces and increases the step distance sufficiently to compensate for the pull (or push) and maintain a relatively constant walking speed in the desired direction. Figure **??** shows velocity plots for Equation 8 demonstrating the relatively robust and controlled nature of the formula.

## IV. Implementation and Results

We highlight the unique aspects of our approach by applying it to numerous tasks to generate fundamental character motion information. The simulations were performed on a machine with the following specifications: Windows7 64-bit, 16 GB Memory, Intel i7-2600 3.4Ghz CPU and compiled and tested with Visual Studio 2010. The simulations used a fixed frame time-step of 0.01s, with a mass of 70kg, and a default leg-length of 1.0m.

### A. *Walking and Standing*

The task of standing or walking (i.e., following a path) is central for any upright biped. Furthermore, for a robust system, the task has to be able to handle disturbances (e.g., from numerical errors and push forces). Hence, we performed numerous simulations to demonstrate how both the spring-damper and impulse-based inverted pendulum model handled these problems. We demonstrate how both models could regain balance through reactive stepping to remedy the disturbance and gradually return to the task at hand (e.g., standing or following a path).

### B. *Complex Uneven Terrain (e.g., Bridge)*

We demonstrate how our impulse-based inverted pendulum model can solve the challenging problem of walking across an unstable terrain, such as a bridge. Furthermore, to make it more interesting and interactive we also applied random force disturbances on single frames to the center-of-mass to interrupt its attempts to balance and walk. However, the unsteady ground and push disturbances did not stop our model from eventually regaining its balance after numerous corrective steps then continuing on its journey.

The impulse performs two tasks; firstly, the impulses emulate the character's rigid leg constraints and keep the character upright; secondly, the impulse applies forces to the bridge rigid body planks. The bridge rigid bodies were held in place using a spring-damper configuration. The inverted pendulum has the ability to handle disturbances from unforeseen push forces, which is accomplished by means of corrective steps.

Furthermore, the bridge did not allow for foot friction (i.e., slipping), and it was necessary to ensure the bridge boards did not displace too far or oscillate out of control. The bridge was kept relatively stiff by choosing suitably large spring-damper constants. Furthermore, if too larger a disturbance or disruption to the ground happened the model corrective step would be outside the bridge's boundary or too large to prevent it falling over.

### C. Limitations

While the IP provides essential stepping information, it has a number of limitations. To summarize:

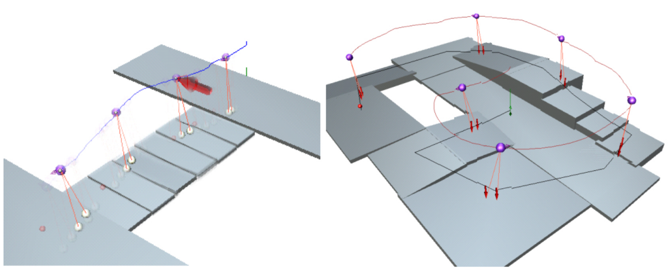
- No upper-body information (pelvis orientation, arms, head)
- Lacks behavioral style (e.g., lazy, happy)

## V. Discussion and Future Work

We have only scratched the surface of the possibilities of generating dynamic character motions. Our model is limited to upright movement and would need to incorporate a much wider repertoire of controllers and behaviors before it is a viable solution for current interactive user environments. However, further work of combining data-driven animations with our system using ad-hoc methods would enable a practical solution with a wide range of behaviors (e.g., get-up, climb, punch). For the impulse coefficient of restitution, we used a value of zero so the character's feet did not bounce. However, for dynamic situations, for example, running or taking long strides (i.e., flight dynamics), we could use values greater than one to emulate the responsive bounce.

## VI. Conclusions

This paper has presented and demonstrated an algorithmically simple and computationally fast method of generating dynamic upright biped character stepping motions. We exploited the IP model and combined it with an impulse-based approach to produce a practical technique for generating fundamental information (i.e., CoM and foot placement data) to reconstruct a character's full body movements. We showed the character IP model responding to unpredictable disturbances and taking intelligent corrective footstep measures to remain upright and balanced. Finally, our character model was able to handle unstable terrain, for example, its foot position moving and changing as it attempted to walk across a bridge.



(a) (b)

Figure 4. **Walking on Complex Terrain** - The impulse inverted pendulum model following a path on varying height terrain.

## References


[1] P. Faloutsos, M. van de Panne, and D. Terzopoulos, "Composable controllers for physics-based character animation," in *Proceedings of the 28th annual conference on Computer graphics and interactive techniques*, no. 1, pp. 251–260, ACM, 2001. 1, 2

[2] T. Komura, H. Leung, and J. Kuffner, "Animating reactive motions for biped locomotion," *In VRST 04: Proceedings of the ACM symposium on Virtual reality software and technology, ACM Press, New York, NY, USA*, pp. 32–40, 2004. 1, 2

[3] A. Macchietto, V. Zordan, and C. R. Shelton, "Momentum control for balance," *ACM Transactions on Graphics*, vol. 28, no. 3 (July), p. 80, 2009. 1

[4] I. Mordatch and M. D. Lasa, "Robust physics-based locomotion using low-dimensional planning," *ACM Transactions on Graphics*, 2010. 1, 2

[5] B. Kenwright, "Responsive biped character stepping: When push comes to shove," in *Cyberworlds (CW), 2012 International Conference on*, pp. 151–156, 2012. 1, 2

[6] S. Kajita, O. Matsumoto, and M. Saigo, "Real-time 3D walking pattern generation for a biped robot with telescopic legs," *Proceedings 2001 ICRA. IEEE International Conference on Robotics and Automation*, pp. 2299–2306, 2001. 1

[7] Y. Abe, M. D. Silva, and J. Popović, "Multiobjective control with frictional contacts," *In Symposium on Computer Animation (SCA)*, pp. 249–258, 2007. 1

[8] K. Sok and M. Kim, "Simulating biped behaviors from human motion data," *ACM Transactions on Graphics (TOG)*, 2007. 1

[9] K. Yin, K. Loken, and M. van de Panne, "Simbicon: Simple biped locomotion control," *ACM Transactions on Graphics (TOG)*, vol. 26, no. 3, p. 105, 2007. 1

[10] M. da Silva, Y. Abe, and J. Popović, "Interactive simulation of stylized human locomotion," *ACM Transactions on Graphics*, vol. 27, p. 1, Aug. 2008. 1, 2

[11] U. Muico, Y. Lee, and J. Popović, "Contact-aware nonlinear control of dynamic characters," *ACM Transactions on Graphics*, 2009. 1

[12] A. J. Stewart and J. F. Cremer, "Beyond keyframing: An algorithmic approach to animation other work in robotics," *Computer Animation*, pp. 273–281, 1992. 1

[13] A. Stewart, "Animation of 3d human locomotion: Climbing stairs and descending stairs," *3rd Eurographics workshop on Animation*, pp. 1–18, 1992. 1

[14] O. Arikan, D. a. Forsyth, and J. F. O'Brien, "Pushing people around," *Proceedings of the 2005 ACM SIGGRAPH/Eurographics symposium on Computer animation*, no. July, pp. 59–66, 2005. 2

[15] K. Yin and D. Pai, "Data-driven interactive balancing behaviors," *Pacific Graphics*, pp. 1–9, 2005. 2

[16] M. V. de Panne, "From footprints to animation," *Computer Graphics Forum*, vol. 16, pp. 211–223, 1997. 2

[17] S. Jain, Y. Ye, and C. K. Liu, "Optimization-based interactive motion synthesis," *ACM Transactions on Graphics*, vol. 28, pp. 1–12, Jan. 2009. 2

[18] T. Komura and K. Ikeuchi, "Stepping motion for a human-like character to maintain balance against large perturbations," *Proceedings 2006 IEEE International Conference on Robotics and Automation, 2006. ICRA 2006.*, no. 1, pp. 2661–2666, 2006. 2

[19] S. Kajita, T. Nagasaki, and K. Kaneko, "A hop towards running humanoid biped," *and Automation, 2004*, no. April, pp. 629–635, 2004. 2

[20] T. Sugihara, "Simulated regulator to synthesize ZMP manipulation and foot location for autonomous control of biped robots," *Robotics and Automation, 2008. ICRA 2008. IEEE*, 2008. 2

[21] M. Popovic and A. Hofmann, "Angular momentum regulation during human walking: biomechanics and control," *Robotics and Automation,*, no. April, pp. 2405–2411, 2004. 2

[22] T. Shiratori, B. Coley, R. Cham, and J. K. Hodgins, "Simulating balance recovery responses to trips based on biomechanical principles," *Proceedings of the 2009 ACM SIGGRAPH/Eurographics Symposium on Computer Animation - SCA '09*, pp. 37–46, 2009. 2

[23] Hsiao T Elizabeth and R. N. Stephen, "Biomechanical influences on balance recovery by stepping," *Journal of Biomechanics*, vol. 32, no. 10, pp. 1099–1106, 1999. 2

[24] J. Pratt, "Velocity-based stability margins for fast bipedal walking," *Fast Motions in Biomechanics and Robotics*, pp. 299–324, 2006. 2

[25] J. Pratt, J. Carff, S. Drakunov, and A. Goswami, "Capture point: A step toward humanoid push recovery," in *Humanoid Robots, 2006 6th IEEE-RAS International Conference on*, pp. 200–207, IEEE, 2006. 2

[26] B. Kenwright, R. Davison, and G. Morgan, "Dynamic Balancing and Walking for Real-Time 3D Characters," *Motion in Games*, pp. 63–73, 2011. 2

[27] C. Wu and V. Zordan, "Goal-directed stepping with momentum control," *In ACM SIGGRAPH/Eurographics Symp. on Computer Animation (2010)*, pp. 113–118, 2010. 2

[28] S. Coros and P. Beaudoin, "Generalized biped walking control," *ACM Transactions on Graphics*, pp. 1–9, 2010. 2, 3

[29] Y.-Y. Tsai, W.-C. Lin, K. B. Cheng, J. Lee, and T.-Y. Lee, "Real-time physics-based 3D biped character animation using an inverted pendulum model.," *IEEE transactions on visualization and computer graphics*, vol. 16, no. 2, pp. 325–37, 2010. 2, 3

[30] B. Mirtich, *Impulse-based dynamic simulation of rigid body systems*. PhD thesis, University of California, Berkeley, 1996. 2, 3

[31] J. Bender and A. Schmitt, "Constraint-based collision and contact handling using impulses," *In Proceedings of the 19th international conference on computer animation and social agents (Geneva, Switzerland)*, no. July, pp. 3–11, 2006. 2

[32] D. Baraff, "Dynamic simulation of non-penetrating rigid bodies," *PhD thesis, Ithaca, NY, USA*, 1992. 3